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Extended MQCD and SUSY/non-SUSY duality

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Abstract: We study the SUSY/non-SUSY duality proposed by Aganagic et al. from Type IIA string and M-theory perspectives. We find that our brane configuration generalizes the so-called *extended* Seiberg-Witten theory on the one hand, and provides a way to realize non-SUSY vacua by intersecting NS5-branes on the other hand. We also argue how the partial SUSY breaking from $\mathcal{N} = 2$ down to $\mathcal{N} = 1$ can be clearly visualized through the brane picture.

Keywords: Supersymmetry Breaking, Intersecting branes models, String Duality, M-Theory.

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Contents

1. Introduction

Recently, Aganagic et al. proposed a SUSY/non-SUSY duality [1] in Type IIB string compactification. In contrast to previous works [2 − 4] where anti-branes are introduced by hand, the breakthrough is to turn on a holomorphic varying background NS-flux H_0 through the non-compact Calabi-Yau (CY) three-fold. This soon suggests a way to realize various kinds of SUSY or non-SUSY vacua via adjusting parameters the NS-flux contains.[1]

Let us briefly review their ideas. Because of the flux $H_0 = dB_0$, four-dimensional gauge theory, realized by wrapping D5-branes on vanishing two-cycles of a CY, acquires different gauge couplings at each $\mathbb{P}^1$ locus:

$$\alpha = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2_{\rm YM}} = \int_{\mathbb{P}^1} B_0(v), \qquad B_0 = B_{RR} + \frac{i}{g_s} B_{\rm NS}, \tag{1.1}$$

where v parameterizes a CY bearing, say, the A_1-type singularity as

$$X : uz + w^2 - W'(v)^2 = 0. \tag{1.2}$$

Note that $W(v) = \sum_{k=1}^{n+1} a_k v^k$, providing a non-trivial A_1 fibration over v, corresponds to the tree-level superpotential breaking $\mathcal{N}=2$ down to $\mathcal{N}=1$. Also, the adjoint chiral field Φ on D5-branes gets identified with the transverse v-direction. Although generalizing X to other ALE fibrations can be carried out, the above prototype will prove to be sufficiently good due to arbitrarily many degrees of freedom inside $B_0(v)$.

[1] Applications and generalizations of these flux vacua are also discussed in a recent paper [5].

The proposed SUSY/non-SUSY duality is achieved by tuning coefficients of the v-dependent background B-field, which has the following expression[2]

$$\mathcal{F}''_{UV}(v) = B_0(v) = \sum_{k=0}^{n-1} t_k v^k, \tag{1.3}$$

where $\mathcal{F}_{UV}(v)$ denotes the ultraviolet prepotential.[3] For generic t_k, SUSY is spontaneously broken at UV. This is accounted for by (1.1), in which one observes that $\mathbb{P}^1$'s may develop relatively different orientations at critical points $W'(v) = \prod_{i=1}^{n}(v - v_i) = 0$ for $\int B_{\rm NS} \sim$ Kähler moduli of $\mathbb{P}^1$. On the other hand, some specific choice of t_k can still make four supercharges preserved, i.e. all orientations of $\mathbb{P}^1$'s are kept aligned. As shown in [1], through geometric transition to dual CY manifolds, SUSY breaking effects can as well be captured qualitatively by studying strongly-coupled IR physics. Minimizing the effective superpotential there, one can further determine t_k from a_k.[4]

Like the brane realization [6 − 8] of meta-stable SUSY breaking vacua [9], our purpose in this paper is to translate things considered above into Type IIA/M-theory language. It is well-known that via a T-duality acting on X one instead obtains two NS5-branes in flat spacetime with D4-branes in between them. From the tree-level F-term

$$\int d^2\theta \; \mathcal{F}''_{UV}(\Phi)\mathcal{W}_\alpha\mathcal{W}^\alpha + W(\Phi), \tag{1.4}$$

one can choose a vacuum $\Phi = diag(v_1, \cdots, v_2, \cdots, \cdots, v_n, \cdots)$ such that the gauge group $\mathrm{U}(N)$ is broken to $\prod_{i=1}^{n}\mathrm{U}(N_i)$. Then, it is seen that D4-branes, coming from fractional D3-branes, remain at v_i's. The size and orientation of $\mathbb{P}^1_i$ controlled by (1.3) are translated, respectively, to the length along the T-dual direction (bare gauge coupling) and sign of RR charge of i-th stack of D4-branes. Based on this Type IIA tree-level description,[5] $B_{\rm NS}(v) < 0$ which naively means negative gauge couplings can be understood as two crossing NS5-branes that result in anti-branes. How spontaneously SUSY breaking vacua occur can therefore be visualized clearly in the presence of both D4- and $\overline{\text{D4}}$-branes as a consequence of the *extended* prepotential.

The rest of this paper is organized as follows. In the next section, we review some known facts about Type IIA/M-theory brane configurations. In section 3, we study the SUSY/non-SUSY duality by introducing a varying B-field. We also comment on the partial SUSY breaking mechanism in terms of Type IIA brane pictures. Finally, we conclude in section 4.

[2] As noted in [1], the degree of $B_0(v)$ polynomial is restricted to at most $n-1$ for triviality of the operator $\mathrm{Tr}\left(\Phi^k W'(\Phi)\mathcal{W}^\alpha\mathcal{W}_\alpha\right)$ in $\mathcal{N}=1$ gauge theory chiral ring.

[3] In $\mathcal{N}=2$ gauge theory, the bare coupling constant $\alpha(\Phi)$ is determined by a holomorphic function $\mathcal{F}_{UV}$ as $\alpha(\Phi) = \mathcal{F}''_{UV}(\Phi)$.

[4] This fact can be interpreted from the M-theory perspective, see below.

[5] As usual, we notice that tree-level field theory results match with classical brane pictures at the lowest order in ℓ_s under $g_s \to 0$, i.e. brane bending and string interaction are not taken into account.

	0123	4	5	6	7	8	9
NS5	∘	∘	∘				
NS5'	∘				∘	∘	
D4	∘			∘			

Table 1: The NS5/D4-brane configuration for fractional D3-branes wrapping the vanishing two-cycle of a conifold after T-duality

2. Type IIA/M-theory brane picture

To set up notations in this paper, we briefly review Type IIA/M-theory brane configurations here.[6]

2.1 Type IIA setup

Viewing alternatively X in (1.2) as an $\mathrm{U}(1)$ fiber over (v, w)-plane, one can go from Type IIB CY geometry to Type IIA Hanany-Witten [11] type brane setup upon a T-daulity along this S^1 (x^6-direction) [12 – 15], namely,

$$(u, z, w, v) \to (\lambda u, \lambda^{-1} z, w, v), \qquad \lambda \in \mathbb{C}^*. \tag{2.1}$$

Note that from now on our convention will be

$$v = x^4 + ix^5, \qquad w = x^7 + ix^8. \tag{2.2}$$

To be precise, take a conifold

$$uz - wv = 0 \tag{2.3}$$

for example. By replacing the conifold tip with a $\mathbb{P}^1$, its A_1 singularity can be treated as if there is a two-center Taub-NUT space. Upon the well-known Taub-NUT/NS5 duality, T-dualizing along the Kaluza-Klein circle x^6 makes the geometry change to two perpendicular NS5-branes shown in table 1. In addition, a complex separation $\Delta x^6 + i\Delta x^9$ arises due to the size of $\mathbb{P}^1$. The vanishing two-cycle assumption enables us to set $\Delta x^9 = 0$.

As far as X concerned, near each critical point where $W'(v_i) = 0$, the geometry locally looks like a conifold. With a $\mathbb{P}^1$ resolution on each singularity, after T-duality, two NS5-branes having common 0123 directions are represented as $w = \pm W'(v)$ on (v, w)-plane and separated along x^6 by l. Furthermore, since D5-branes wrapping vanishing two-cycles now become D4-branes extending along 01236, the effective four-dimensional gauge coupling reads

$$\frac{1}{g_{\rm YM}^2} = \frac{l}{8\pi^2 g_s \ell_s} = \frac{1}{4\pi g_s} \int_{\mathbb{P}^1} B_{\rm NS}. \tag{2.4}$$

The second equality reveals how the Kähler moduli of $\mathbb{P}^1$ is related to Δx^6 separation.

[6] For more details, see [10] and references therein.

2.2 M-theory lift

To study the corresponding IR physics, Witten suggested that one should take both large l and $R_{10} = g_s \ell_s$ limit in (2.4) with $\frac{1}{g^2_{\rm YM}}$ being kept finite. This means that the M-cycle opens up and Type IIA branes are unified by one smooth M5-brane [16]. Besides, four-dimensional gauge theory will now be characterized by long-distance informations on the M5-brane.

In the case without t_k perturbation, except for 0123, the M5-brane wraps a complex curve Σ, holomorphically embedded in $\mathcal{M}_6$ ($x^{4,5,6,8,9}$ plus the M-cycle x^{10}) and parameterized by $(w(v), t(v))$ with $t = e^{-s} = \exp -R_{10}^{-1}(x^6+ix^{10})$. Σ becomes either a Seiberg-Witten curve on (v,t)-plane or a planar loop equation on (v,w)-plane, see figure 1. More precisely, a hyperelliptic curve

$$w^2 - W'(v)w + f_{n-1}(v) = 0, \tag{2.5}$$

of genus $g = n-1$ on (v,w)-plane, which approaches asymptotically to $w = W'(v)$ and $w = 0$ at $|v| \to \infty$, stands for the underlying planar loop equation of $\mathcal{N}=1$ Dijkgraaf-Vafa matrix model [17].

On the other hand, a degenerated Seiberg-Witten curve $t^2 + P_N(v)t + \Lambda^{2N} = 0$ (Λ: dynamical scale), which implies that $N-(g+1)$ mutually local massless monopoles appear, is seen on (v,t)-plane. That is, the discriminant now factorizes into

$$\begin{aligned} \triangle_{\rm SW} &= P_N(v)^2 - 4\Lambda^{2N} = H_{N-n}(v)^2 F_{2n}(v), \\ P_N(v) &= \langle \det(v-\Phi) \rangle, \end{aligned} \tag{2.6}$$

where H_{N-n} and F_{2n} are polynomials with simple zeros of degrees $N-n$ and $2n$, respectively. It is found [18] that the extremized M-theory curve gives rise to a relation

$$P_N(v)^2 - 4\Lambda^{2N} = \big(W'(v)^2 - f_{n-1}(v)\big) H_{N-n}(v)^2 \tag{2.7}$$

between (2.6) and $\mathcal{N}=1$ planar loop equation under the constraint

$$P_N(v) \to \prod_{i=1}^{n} (v-v_i)^{N_i}, \qquad \sum_{i=1}^{n} N_i = N, \qquad \text{as} \qquad \Lambda \to 0. \tag{2.8}$$

The uniqueness of $P_N(v)$ in (2.7) determines coefficients of the polynomial $f_{n-1}(v)$ such that all glueball vevs in turn get fixed. In fact, there is a parallel in the presence of t_k in (1.3). As argued in [1], parameters t_k and a_k, concerning the shape of Σ, are not independent but related to each other at IR. Similarly, this is because an on-shell M5-brane has to have its volume minimized (minimization of the glueball superpotential).

3. SUSY/non-SUSY duality

Let us now turn on arbitrary t_k inside B-field such that each group of D4-branes in between NS5-branes will no longer have equal length. Their lengths vary according to

$$l(v) = 2\pi \ell_s \int_{\mathbb{P}^1} B_{\rm NS}(v). \tag{3.1}$$

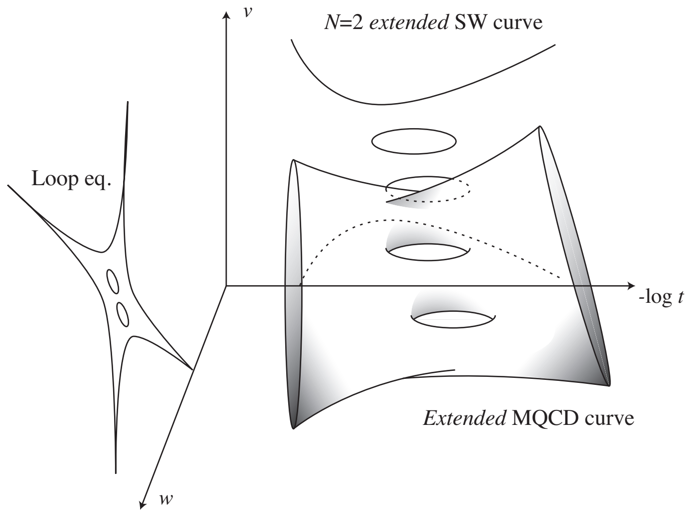

Figure 1: The *extended* $\mathcal{N}=1$ MQCD curve in (v,w,t)-space. The projection onto (v,t)- and (v,w)-plane represents the degenerated *extended* SW curve and planar loop equation, respectively.

If the tree-level superpotential $W(v)$ is dropped out, one is left with the so-called $\mathcal{N}=2$ *extended* Seiberg-Witten theory [19] whose UV Lagrangian is

$$\mathcal{L}_{UV} = \frac{1}{2\pi}\mathrm{Im}\,\mathrm{Tr}\left[\int d^4\theta\, \mathcal{F}'_{UV}(\Phi)e^V\bar{\Phi} + \int d^2\theta\, \frac{1}{2}\mathcal{F}''_{UV}(\Phi)\mathcal{W}_\alpha\mathcal{W}^\alpha\right], \tag{3.2}$$

where V is the $\mathcal{N}=1$ vector superfield and $\mathcal{F}_{UV}(\Phi)$ as in (1.3) contains higher Casimir terms.

To be explicit, an example with the following prepotential and superpotential

$$\begin{aligned} \mathcal{F}_{UV}(\Phi) &= \mathrm{Tr}\left(\frac{t_2}{12}\Phi^4 + \frac{t_1}{6}\Phi^3 + \frac{t_0}{2}\Phi^2\right), \\ W(\Phi) &= \mathrm{Tr}\left(a_4\Phi^4 + a_3\Phi^3 + a_2\Phi^2 + a_1\Phi\right), \end{aligned} \tag{3.3}$$

is plotted in figure 2. In spite of t_k, the singular CY geometry can still be read off from (v,w)-plane projection, i.e. $w\big(w - W'(v)\big) = 0$. However, D4-branes are no longer equally-spaced on (v,x^6)-plane but stretch over the interval

$$\Delta x_6 = l(v_i) \propto \mathcal{F}''_{UV}(v_i) \tag{3.4}$$

for i-th gauge factor. For usual $\mathcal{N}=2$ SU(N) SW theory, which is asymptotically free, the inverse gauge coupling has a logarithmic one-loop correction. This fact is reflected on the bending of the MQCD curve, i.e. $t \sim v^N$ for large v and t. In our case (3.3), asymptotically we expect that the bending includes an extra quadratic term v^2, see figure 3.

Our classical Type IIA brane configuration is new in the sense that not only (v,w) but (v,x^6) projection yields relatively curved NS5-branes. Besides, using these brane setups,

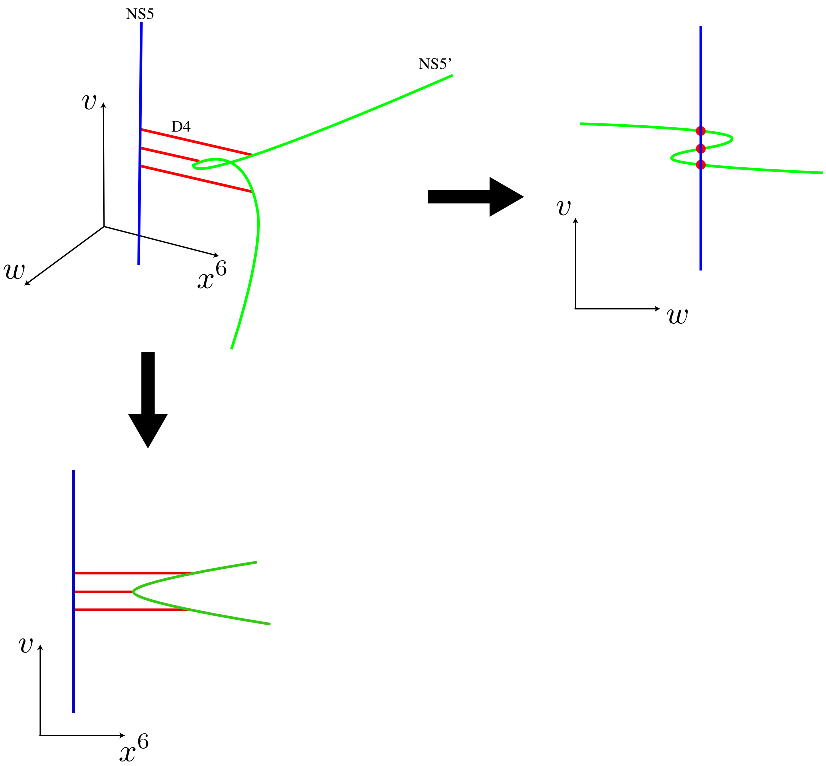

Figure 2: A classical SUSY vacuum in terms of the Type IIA brane picture. Two NS5-branes have D4-branes distributed at critical loci. Here, 01239 directions are suppressed. The down arrow indicates that a varying B-field results in differently-sized D4-branes and provides an UV setup for the *extended* Seiberg-Witten theory. The right arrow implies that, despite t_k deformation, the underlying CY geometry is still encoded rightly on (v, x^6)-plane.

one can easily judge whether it preserves SUSY or not from the intersection of NS5- and D4-branes on (v, x^6)-plane. This is illustrated in figure 4.

We have assumed a linear $\mathcal{F}''_{UV}$ and a quadratic W' just as in [1]. The authors there showed that how SUSY and non-SUSY vacua occur according to $\mathcal{F}_{UV}$. The difference between figure 4 (a) and (b) lies on a displacement along x^6, namely, the value of t_0 in (1.3). Naively, the lower stack of D4-branes in figure 4 (b) acquires a negative bare gauge coupling for $\Delta x^6 \propto -\frac{1}{g^2_{\rm YM}} < 0$ as argued in [1]. Rather, this can be interpreted as the presence of anti-branes or, in Type IIB language, the flip of orientations of $\mathbb{P}^1$'s. When lifted to M-theory such that [16]

$$s(v) = \Delta x^6 + i\Delta x^{10} \propto \frac{4\pi}{g^2_{\rm YM}} + i\frac{\theta}{2\pi}, \tag{3.5}$$

the above fact then emphasizes that the M5-brane can no more stay supersymmetric due to its non-holomorphic way of embedding with both s and $\bar{s}$. As far as the matrix model spectral curve (2.5) concerned, anti-eigenvalues (holes) dwelling in $W'(v) = 0$ [17] can be thought of as the appearance of anti-branes in (v, x^6)-space, which do not disturb what happen in (v, w)-space.

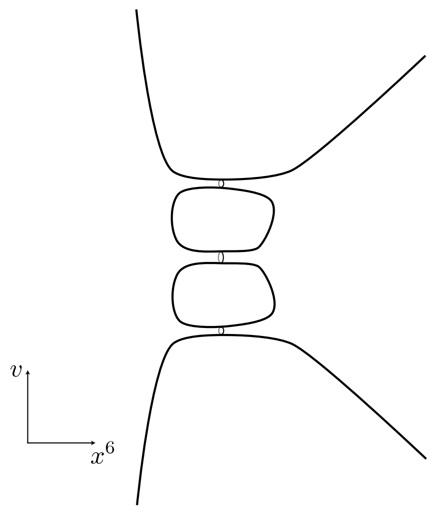

Figure 3: The degenerated *extended* SW curve on (v, x^6)-plane. The shape of r.h.s. NS5-brane is asymptotically $s \sim v^2 + N \log v$ at large v because of the quartic prepotential in (3.3).

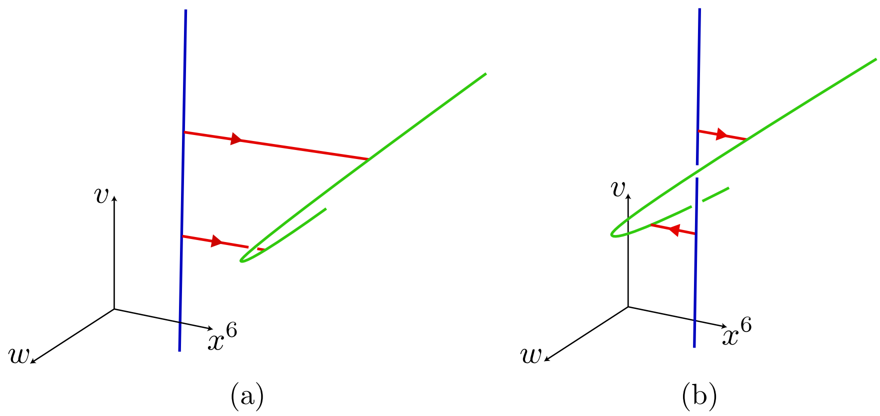

Figure 4: Classical Type IIA configurations for (a) SUSY and (b) non-SUSY phases. In the non-SUSY case, the orientation of D4-branes is flipped. (a) and (b) differ only by a displacement along x^6.

By doing so, spontaneously SUSY breaking vacua can be explicitly constructed by means of Type IIA brane configurations like figure 4. The terminology "SUSY/non-SUSY duality" bears similarity to Seiberg duality because they amount to crossing NS5-branes and thereby changing the coupling constant.

3.1 $\mathcal{N} = 1$ effective superpotential

Now, let us see how the effective superpotential gets modified in the presence of t_k. For $\mathcal{N} = 1$ gauge theory, the M-theory approach to deriving the effective superpotential is initiated by Witten [20]. He suggested the following integral

$$W_{\mathrm{MQCD}} = \int_B \Omega_3, \tag{3.6}$$

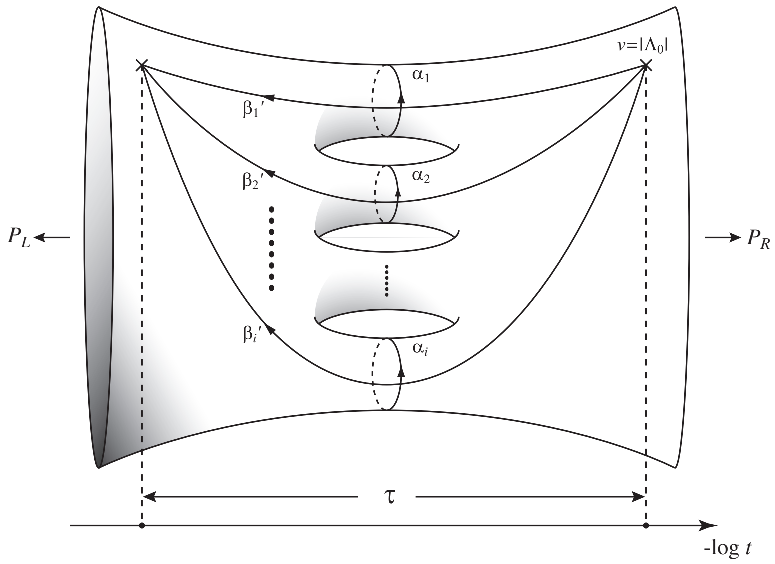

Figure 5: Cycles for integrals in the glueball superpotential. β_i'-cycles are regulated at $|v| = \Lambda_0$.

where $\Omega_3 \equiv dv \wedge dw \wedge \frac{dt}{t}$ is a holomorphic three-form. B is a three-manifold having two boundaries, i.e. the previously-defined Σ and a reference surface Σ_0 homologous to Σ.

If there exists a two-form Ω_2 which satisfies

$$\Omega_3 = d\Omega_2, \tag{3.7}$$

then (3.6) can be written as

$$W_{\mathrm{MQCD}} = W(\Sigma) - W(\Sigma_0), \tag{3.8}$$

where $W(\Sigma) = \int_\Sigma \Omega_2$ and $W(\Sigma_0) = \int_{\Sigma_0} \Omega_2$. Since $W(\Sigma_0)$ is physically irrelevant, the effective superpotential reduces to

$$W_{\mathrm{eff}} = \int_\Sigma \Omega_2, \qquad \Omega_2 = -w dv \wedge \frac{dt}{t}. \tag{3.9}$$

Now, it is straightforward that

$$W_{\mathrm{eff}} = -\int_\Sigma w dv \wedge \frac{dt}{t} = \sum_i \left(\oint_{\alpha_i} \frac{dt}{t} \int_{\beta_i} w dv - \int_{\beta_i} \frac{dt}{t} \oint_{\alpha_i} w dv \right) \tag{3.10}$$

upon making use of Riemann's bilinear identity. Here, α_i's denote cycles around cuts while β_i's are paths connecting P_R and P_L, see figure 5.

Because $t \sim v^{N_i}$ near the neighborhood of each cut (N_i: number of D4-branes attached on the cut), one has

$$\oint_{\alpha_i} \frac{dt}{t} = N_i. \tag{3.11}$$

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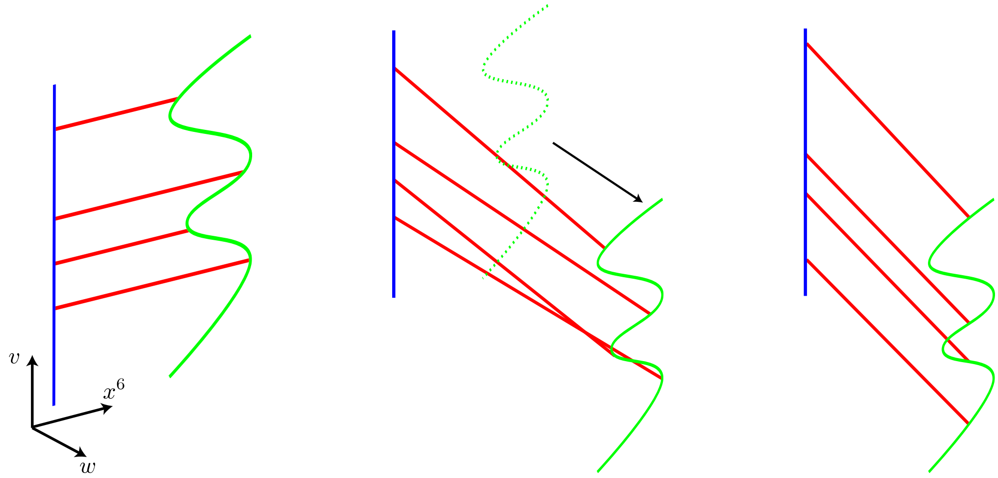

(a) (b) (c)

Figure 6: Partial SUSY breaking from $\mathcal{N} = 2$ configuration to $\mathcal{N} = 1$ one. Turning on FI parameters, SUSY of the *extended* $\mathcal{N} = 2$ theory gets completely broken (off-shell) temporarily. SUSY is recovered (on-shell) again at critical loci where $\mathcal{F}''(\Phi) = W'(\Phi) = 0$, but only $\mathcal{N} = 1$ is now preserved.

Next, since integrals over β_i naively diverge, it is necessary to introduce a cut-off scale at $|v| = \Lambda_0$ for regularization. The integral $\int_{\beta'} \frac{dt}{t}$ is nothing but a line integral over the coordinate s, that is, it just gives a regularized complex separation between two NS5-branes or the gauge coupling on the compactified D4-brane. Therefore, the bare Yang-Mills coupling constant $\alpha_i(\Lambda_0) = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\rm YM}^2}$ evaluated at Λ_0 is related to $\tau_i = \int_{\beta_i'} \frac{dt}{t}$ by

$$\frac{\tau_i}{2\pi i R_{10}} = -\alpha_i, \qquad R_{10} = g_s \ell_s. \tag{3.12}$$

Plugging these into (3.10), we obtain the effective superpotential ($R_{10} = 1$)

$$W_{\rm eff} = \sum_i \left(N_i \Pi_i + 2\pi i \alpha_i S_i\right), \tag{3.13}$$

where the glueball $S_i \equiv \oint_{\alpha_i} w dv$ and $\frac{\partial \mathcal{F}_0}{\partial S_i} = \Pi_i \equiv \int_{\beta_i'} w dv$ stand for dual periods in the context of special geometry. With t_k perturbation, from (3.4) we find that (3.10) can be immediately generalized into

$$W_{\rm eff} = \sum_i \big(N_i \Pi_i + 2\pi i \oint_{\alpha_i} \alpha_i(v) w(v) dv\big). \tag{3.14}$$

This reproduces precisely what derived by Aganagic et al. in [1].

3.2 Partial SUSY breaking from $\mathcal{N} = 2$ to $\mathcal{N} = 1$

Finally, we comment on the partial SUSY breaking from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ through the above brane picture. The partial SUSY breaking is discussed in [21 – 23] for Abelian gauge group. The non-Abelian generalization is well investigated and established by authors

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of [24]. According to these early works, $\mathcal{N}=2$ theory is firstly perturbed by introducing a general prepotential of the form (1.3) and its SUSY is broken down to $\mathcal{N}=1$ thereof upon adding Fayet-Iliopoulos (FI) parameters.

As mentioned throughout this paper, the prepotential $\mathcal{F}_{UV}(\Phi)$ describes how NS5-branes get deformed in (v, x^6)-space, see figure 6 (a). Three FI parameters correspond to the relative position of two NS5-branes in (x^7, x^8, x^9)-space. Henceforth, turning on FI parameters means that two NS5-branes are separated from each other in (x^7, x^8, x^9)-space. In addition, a new direction by which the bare coupling constant is measured should be defined due to the presence of FI parameters. In figure 6 (a), initial positions of D4-branes are not fixed. But if D4-branes still remain at their initial positions, they become non-parallel when the curved NS5-brane is pulled along w, see figure 6 (b). In other words, SUSY can no longer be maintained for an off-shell choice of Φ vev. To recover SUSY, D4-branes should be re-distributed appropriately at critical loci, i.e. $\mathcal{F}''(\Phi)=0$ (on-shell condition). Now, since SUSY is recovered to $\mathcal{N}=1$, we can as well recognize the tree-level superpotential as $W(\Phi)=\mathcal{F}'(\Phi)$ with coefficients rescaled suitably, see figure 6 (c). To this end, the partial SUSY breaking mechanism can thus be understood pictorially from the *extended* brane configuration.

The above argument is valid only for the classical (UV) theory. In order to extend this picture to the full quantum theory, we need to replace the NS5/D4 system with a MQCD curve. Pulling out one NS5-brane then corresponds to deforming the curve. The projective information of the MQCD curve contains both the *extended* SW curve and loop equation (or, alternatively, generalized Konishi anomaly equation) of $\mathcal{N}=1$ theory. Therefore, it is interesting to see how these aspects transform according to the partial SUSY breaking deformation of the curve.

4. Conclusion and discussion

So far, we have shown how the SUSY/non-SUSY duality proposed by Aganagic et al. can have a corresponding Type IIA brane picture. Apart from conventional ones, where anti-branes are wrapped on a CY by hand, the setup here involves changing the orientation of local two-cycles through a varying background NS-flux. This dose work because B-field gives a Kähler moduli $\Delta t \sim B_{\mathrm{NS}}$ of arbitrary sign to shrinking two-cycles and hence controls their flops. On Type IIA side, we interpret this background as two crossing NS5-branes where $\overline{\mathrm{D4}}$-branes appear naturally for flipped orientations. Consequently, simultaneous presence of D4- and $\overline{\mathrm{D4}}$-branes soon suggests a way to realize various kinds of SUSY/non-SUSY vacua via adjusting parameters the NS-flux contains. Moreover, curved NS5-branes on (v,t)-plane with $\mathcal{N}=2$ SUSY correspond to what has been known as the *extended* Seiberg-Witten theory. One can further add FI parameters to partially break $\mathcal{N}=2$ down to $\mathcal{N}=1$. Resorting to Type IIA brane pictures, we see this process is clearly visualized in figure 6. The final $\mathcal{N}=1$ vacuum is arrived at once the tree-level superpotential $W(\Phi)$ takes the form of $\mathcal{F}'(\Phi)$.

We also considered M-theory lift. Without t_k perturbation, the M-theory curve itself is either a degenerated Seiberg-Witten curve on (v,t)-plane or a loop equation of DV matrix

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model on (v, w)-plane. Though adding higher t_k terms has no effect on the planar loop equation, we find that $\mathcal{N} = 1$ effective superpotential which involves β-cycles on (v, t)-plane gets modified. In particular, it seems that the above partial SUSY breaking process can be described by deforming one given M-theory curve in order to incorporate quantum effects. It is thus of interest to compare this observation with field theory results found in [25]. We leave these problems to future works.

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